CONTENTS

MENSURATION

Circle

Area of a circle, radius r is πr^2
Circumference of circle is $2\pi r$

Sphere

Volume of a sphere, radius r, is $\frac{4}{3}\pi r^3$
Surface area of sphere is $4\pi r^2$

Right circular cylinder

Volume of cylinder, radius r and height h is $\pi r^2 h$
Curved surface area is $2\pi rh$

Right circular cone

Volume of cone, radius r, and height h is $\frac{1}{3}\pi r^2 h$
Curved surface area is πrl where l is the slant height of the cone.

ALGEBRA

Factors

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Permutations and Combinations

$${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Finite Series

$$\sum_{q=0}^{n-1} (a + qd) = \frac{n}{2}\,[2a + (n-1)d\,]; \quad \sum_{q=0}^{n-1} ar^q = \frac{a(1-r^n)}{1-r}$$

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1); \quad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1); \quad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \binom{n}{r}x^r + \ldots + x^n \ (n\text{+ve int.})$$

de Moivre's Theorem

If n is an integer, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

If n is a rational number, $\cos n\theta + i\sin n\theta$ is one of the values of $(\cos\theta + i\sin\theta)^n$.

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\sinh^{-1} x = \ln\ [x + \sqrt{(x^2 + 1)}\ ]$$

Principal value of $\cosh^{-1} x = \ln\ [x + \sqrt{(x^2 - 1)}\ ]$ $\qquad (x \geq 1)$

$$\tanh^{-1} x = \tfrac{1}{2} \ln \left| \frac{1 + x}{1 - x} \right| \qquad (|x| < 1)$$

CIRCULAR FUNCTIONS

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\left.\begin{array}{ll} \text{If } \sin\theta = \sin\alpha, & \text{then } \theta = n\pi + (-1)^n\alpha \\ \text{If } \cos\theta = \cos\alpha, & \text{then } \theta = 2n\pi \pm \alpha \\ \text{If } \tan\theta = \tan\alpha, & \text{then } \theta = n\pi + \alpha \end{array}\right\} \text{ where } n = 0, \pm 1, \pm 2, \ldots$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$
$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$
$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$
$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$
$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

If $\tan\frac{A}{2} = t$, then $\sin A = \frac{2t}{1+t^2}$; $\cos A = \frac{1-t^2}{1+t^2}$

COORDINATE GEOMETRY

Perpendicular distance from (h, k) to $ax + by + c = 0$ is $\left|\frac{ah + bk + c}{\sqrt{(a^2 + b^2)}}\right|$

The acute angle between two lines with gradients m_1, m_2 is

$\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Area of Triangle is

$$\left| \tfrac{1}{2} \left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right] \right| = \tfrac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre at $(-g, -f)$ and radius $\sqrt{(g^2 + f^2 - c)}$.

The parametric equations of a circle with centre at (a, b) and radius r are $x = a + r\cos t$, $y = b + r\sin t$.

Point dividing $P_1 P_2$ in the ratio $k : 1$ has coordinates

$$\left(\frac{x_1 + kx_2}{1 + k}, \frac{y_1 + ky_2}{1 + k}, \frac{z_1 + kz_2}{1 + k}\right)$$

Angle ϕ between two lines with direction cosines l, m, n:

l', m', n' is given by $\cos\phi = \dfrac{\pm(ll' + mm' + nn')}{\sqrt{(l^2 + m^2 + n^2)}\sqrt{(l'^2 + m'^2 + n'^2)}}$

Distance from $P_1(x_1, y_1, z_1)$ to plane $Ax + By + Cz + D = 0$ is

$$\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{(A^2 + B^2 + C^2)}}\right|$$

Plane distance p from origin, direction cosines of normal l, m, n,

$$lx + my + nz = p.$$

Line through (x_1, y_1, z_1), direction cosines, l, m, n.

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = t.$$

CALCULUS

I. INFINITE SERIES

Taylor's Theorem

$$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!}f''(a) + \ldots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(a) + \ldots,$$

with 'remainder term' $\frac{x^r}{r!}f^{(r)}(a+\theta x)$, where $0 < \theta < 1$.

Maclaurin's Theorem

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(0) + \ldots,$$

with 'remainder term' $\frac{x^r}{r!}f^{(r)}(\theta x)$, where $0 < \theta < 1$.

$$\exp x \equiv e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad *$$

$$\log_e(1+x) \equiv \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r-1}\frac{x^r}{r} \ldots$$

valid for $-1 < x \leq 1$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad *$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r\frac{x^{2r}}{(2r)!} + \ldots \qquad *$$

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad *$$

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \qquad *$$

* *These series are valid for all finite x.*

II DERIVATIVES

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
e^x	e^x
$a^x (a > o)$	$a^x \ln a$
$\log_e x \equiv \ln x$	$\frac{1}{x}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
uv	$uv' + u'v$
$\frac{u}{v}$	$(vu' - uv')/v^2$

III INTEGRALS *(Constants of integration are omitted;* $\ln a \equiv \log_e a$*)*

$f(x)$	$\int f(x)\,dx$
$\dfrac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{(a^2+x^2)}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{(a^2+x^2)}}$	$\ln\{x+\sqrt{(x^2+a^2)}\}$ or $\sinh^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{x}{\sqrt{(a^2+x^2)}}$	$\sqrt{(a^2+x^2)}$
$\dfrac{1}{\sqrt{(x^2-a^2)}}$	$\ln\{x+\sqrt{(x^2-a^2)}\}$ or $\cosh^{-1}\left(\dfrac{x}{a}\right)$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln(\sec x)$
$\cot x$	$\ln(\sin x)$
$\sec x$	$\ln(\sec x+\tan x)$ or $\ln\left\{\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\}$
$\operatorname{cosec} x$	$\ln\tan\dfrac{x}{2}$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$
$u\dfrac{dv}{dx}$	$uv-\int v\dfrac{du}{dx}\,dx$

IV APPLICATIONS

For a curve $y = f(x)$, $a \leq x \leq b$.

Area between curve and x-axis $= \int_a^b y\,dx$

Mean value $= \frac{1}{b-a}\int_a^b y\,dx$

Volume of revolution about x-axis $= \pi \int_a^b y^2\,dx$

Centroid of area between curve and x-axis has coordinates

$$\bar{x} = \frac{\int_a^b xy\,dx}{\int_a^b y\,dx}; \qquad \bar{y} = \frac{\int_a^b \tfrac{1}{2} y^2\,dx}{\int_a^b y\,dx}$$

Centroid of solid of revolution about x-axis:

$$\bar{x} = \frac{\int_a^b xy^2\,dx}{\int_a^b y^2\,dx}$$

For the area shown in Figure 1

First moment about x-axis $= \int_a^b \frac{1}{2} y^2 \, dx$

First moment about y-axis $= \int_a^b xy \, dx$

Second moment about x-axis $= \int_a^b \frac{1}{3} y^3 \, dx$

Second moment about y-axis $= \int_a^b x^2 y \, dx$

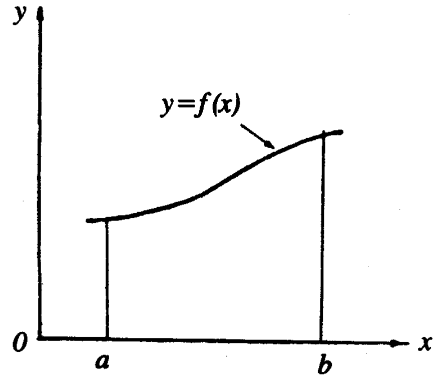

Fig. 1

For the solid of revolution shown in Figure 2

First moment about xy-plane $= 0$

First moment about yz-plane $= \pi \int_a^b xy^2 \, dx$

Second moment about x-axis $= \pi \int_a^b \frac{1}{2} y^4 \, dx$

Second moment about y-axis $= \pi \int_a^b y^2 \left(x^2 + \frac{y^2}{4}\right) dx$

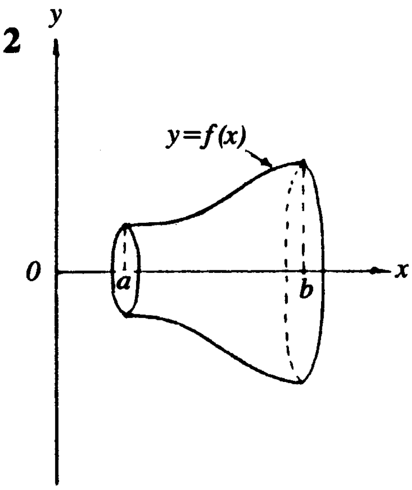

Fig. 2

$$\textit{Length of arc} = \int_a^b \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}\, dx = \int_{t_1}^{t_2} \sqrt{(\dot{x}^2 + \dot{y}^2)}\, dt$$

$$\textit{Area of surface of revolution} = 2\pi \int_a^b y\sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}\, dx$$

$$= 2\pi \int_{t_1}^{t_2} y\sqrt{(\dot{x}^2 + \dot{y}^2)}\, dt$$

$$\textit{Radius of curvature } \rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}}$$

Polar coordinates

$$\textit{Area enclosed by curve} = \tfrac{1}{2} \int_{\theta_1}^{\theta_2} r^2\, d\theta$$

$$\textit{Length of arc} = \int_{\theta_1}^{\theta_2} \sqrt{\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}}\, d\theta = \int_{r_1}^{r_2} \sqrt{\left\{1 + r^2\left(\frac{d\theta}{dr}\right)^2\right\}}\, dr$$

$$\textit{Radius of curvature } \rho = r \Big/ \frac{dp}{dr}$$

V APPROXIMATIONS

Trapezoidal Rule:

$$\int_a^b y\,dx \approx \tfrac{1}{2} h \{(y_o + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$$

Simpson's rule (n even)

$$\int_a^b y\,dx \approx \tfrac{1}{3} h \{(y_o + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1})$$
$$+ 2(y_2 + y_4 + \ldots + y_{n-1})\}$$

Newton's approximation to a rot of $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

VECTORS

Line through point, position vector **a**, parallel to **b**

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

Position vector of a point dividing the line joining P, Q, with position vectors **p**, **q** in the ratio $\lambda:\mu$ is $\dfrac{\lambda\mathbf{q} + \mu\mathbf{p}}{\lambda + \mu}$

Plane through point, position vector **a**, perpendicular to **n**

$$(\mathbf{r} - \mathbf{a}).\ \mathbf{n} = 0$$

Scalar product $= \mathbf{a}_1 . \mathbf{a}_2 = a_1 a_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

Vector product $= \mathbf{a}_1 \times \mathbf{a}_2 = \mathbf{a}_1 \wedge \mathbf{a}_2 = a_1 a_2 \sin\theta\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$

MECHANICS

Centres of mass

Arc, radius r, angle 2θ	$r \sin\theta/\theta$ from centre
Sector of circle, radius r, angle 2θ	$\frac{2}{3}r \sin\theta/\theta$ from centre
Hemisphere, radius r	$\frac{3}{8}r$ from centre
Hemispherical shell, radius r	$\frac{1}{2}r$ from centre
Solid cone, height h	$\frac{1}{4}h$ from vertex
Conical shell, height h	$\frac{2}{3}h$ from vertex

Moments of inertia

Rod, length $2l$, about perpendicular axis through centre	$\frac{1}{3}ml^2$
Disc, radius r, about perpendicular axis through centre	$\frac{1}{2}mr^2$
Hoop, radius r, about a diameter	$\frac{1}{2}mr^2$
Solid sphere, radius r, about diameter	$\frac{2}{5}mr^2$
Spherical shell, radius r, about a diameter	$\frac{2}{3}mr^2$
Parallel axes theorem	$I_A = I_G + M(GA)^2$
Perpendicular axes theorem for a lamina	$I_{oz} = I_{ox} + I_{oy}$

Simple harmonic motion

$$\frac{d^2x}{dt^2} = -\omega^2 x, \left(\frac{dx}{dt}\right)^2 = \omega^2(a^2 - x^2), x = a \sin(\omega t + \epsilon)$$

Compound pendulum

Period $= 2\pi \sqrt{(k^2 + h^2)/gh}$

Components of acceleration

$\ddot{r} - r\dot{\theta}^2$ along radius vector

$2\dot{r}\dot{\theta} + r\ddot{\theta}$ perpendicular to radius vector

PROBABILITY

Probability laws

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B \mid A)$$

Discrete variable X with probability function $P(X = x)$	Continuous variable X with probability density function $f(X)$
Distribution function $F(X)$ $F(x_o) = P(X \le x_o)$ $= \sum_{x \le x_o} P(x)$	$F(x) = P(X < x_o)$ $= \int_{-\infty}^{x_o} f(x)\,dx$
Expectation of X $E(X) = \Sigma x P(X = x)$	$E(X) = \int x f(x)\,dx$
Expectation of $g(x)$ $E[g(X)] = \Sigma g(x) P(X = x)$	$E[g(X)] = \int g(x) f(x)\,dx$
Variance σ^2 $\text{Var}(X) = E[\{X - E(X)\}^2]$	
Covariance $\text{Cov}(X_1, X_2) = E[\{X_1 - E(X_1)\}\{X_2 - E(X_2)\}]$	
Correlation coefficient $\rho_{12}(X_1, X_2)$ $\rho_{12} = \dfrac{\text{Cov}(X_1, X_2)}{\sqrt{\{\text{Var}(X_1)\,\text{Var}(X_2)\}}}$	
Linear regression coefficient, β_{12}, for X_1 on X_2 $\beta_{12} = \dfrac{\text{Cov}(X_1, X_2)}{\text{Var}(X_2)}$	

Probability generating function $G(z)$

$$G(z) = P(0) + P(1)z + P(2)z^2 + \ldots + P(r)z^r + \ldots,$$

where $P(r) = P(X = r)$

Binomial distribution (X, p, N)

$$P(X = k) = \binom{N}{k} p^k (1 - p)^{N-k}$$

$$E(X) = Np$$

$$\text{Var}(X) = Np(1 - p)$$

$$G(z) = [pz + (1 - p)]^N$$

Poisson distribution (X, m)

$$P(X = k) = \frac{e^{-m} m^k}{k!}$$

$$E(X) = m$$

$$\text{Var}(X) = m$$

$$G(z) = e^{-m} e^{mz}$$

Normal distribution

If X is distributed $N(\mu, \sigma)$ then $\frac{X - \mu}{\sigma}$ is distributed $N(0, 1)$.