1. (a) Since $AD$ is parallel to $BC$, then $\overrightarrow{AB} = 2\overrightarrow{DC}$

(b) $\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC} = \overrightarrow{OC} - \overrightarrow{OD} = 5\text{i} + 8\text{j} - (\text{i} + 5\text{j}) = 4\text{i} + 3\text{j}$

(c) $\overrightarrow{AB} = 2\overrightarrow{DC} = 2(4\text{i} + 3\text{j}) = 8\text{i} + 6\text{j}$

(d) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \text{i} + \text{j} + (8\text{i} + 6\text{j}) = 9\text{i} + 7\text{j}$

Hence $B$ is given by $B(9, 7)$.

2. (a) The weight and the tension are the forces acting on $C$.

Resolving vertically, since $C$ is in equilibrium:

$$T \sin 30^\circ + T \sin 30^\circ = 10$$

$$2T \sin 30^\circ = 10$$

$$2T \cdot \frac{1}{2} = 10$$

$$T = 10 \text{ N}$$

(b) The forces acting on ring $A$ are the weight, tension, friction and normal reaction.
(c) With reference to the above diagram, resolving vertically:

\[ R = 10 + T \sin 30^\circ \]

\[ R = 10 + 10 \cdot \frac{1}{2} \]

\[ R = 15\text{N} \]

Resolving horizontally:

\[ F = T \cos 30^\circ \]

\[ F = 10 \cdot \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{N} \]

3. (a) \(P\) and \(Q\) are the vertical and horizontal components of the reaction at the hinge.

(b) Taking moments about \(A\), since the system is in equilibrium:

\[ (70)(25 \cos \theta) = (T)(14) \]

and since \(\cos \theta = \frac{48}{50} = \frac{24}{25}\) then \(T = 120\text{N}\)

(c) System is in equilibrium. Thus resolving vertically:

\[ P = 70\text{N} \]

3
Resolving horizontally:

\[ Q = T \]

\[ Q = 120 \text{N} \]

(d) The Three-Force result states that if a rigid body is in equilibrium under the action of 3 forces then the lines of action of these forces must be either parallel or concur. There are only 3 forces acting on the rod, namely \( R \) (the resultant of \( P \) and \( Q \)), the weight and the tension. The lines of action of the weight and the tension intersect at the mid-point of \( BC \) (so the forces must concur). Hence the reaction \( R \) must also pass through the mid-point of \( BC \).

4. (a) With reference to the diagram above, \( \sin \alpha = \frac{5}{13} \) and \( \cos \alpha = \frac{12}{13} \).

Resolving horizontally:

\[ 25 = Y - 15 + 26 \cos \alpha \]

\[ 25 = Y - 15 + 26 \cdot \frac{12}{13} \]

\[ 25 = Y - 15 + 24 \]

\[ Y = 16 \text{N} \]

Resolving vertically:

\[ -19 = 15 - X - 26 \sin \alpha \]

\[ -19 = 15 - X - 26 \cdot \frac{5}{13} \]

\[ -19 = 15 - X - 10 \]

\[ X = 24 \text{N} \]

(b) For the system of forces, taking moments about \( Q \) in a clockwise direction:
Moment about Q = \((Y \times 5) - (15 \times 12) + (26 \cos \alpha \times 5)\)

\[= 80 - 180 + 120 = 20 \text{Nm}\]

Thus the resultant force must also have a moment of 20Nm (clockwise) about Q.

The line of action of the resultant must cross QR as shown:

```
\[\begin{array}{c}
\text{Q} & \text{R} \\
\text{x} & 25 \text{N} \\
19 \text{N}
\end{array}\]
```

Moment of resultant force about Q = 20Nm

\[19x = 20\]

\[x = \frac{20}{19} \text{ m (to the right of Q)}\]

(c) (i) The resultant force remains the same in either case i.e. \((25i - 19j)\) N

(ii) Moment of system of forces about P = Moment of resultant about P + C

\[(15 \times 5) - (15 \times 12) = 0 + C\]

\[-105 \text{Nm} = C\]

So the Couple has a magnitude of 105Nm in a counterclockwise sense.

5. (a) \(\tan \alpha = \frac{3}{4}\). Thus, \(\sin \alpha = \frac{3}{5}\) and \(\cos \alpha = \frac{4}{5}\).

For object A, resolving parallel to the plane:

\[T = 50 \sin \alpha\]

\[T = 50 \times \frac{3}{5} = 30 \text{ N}\]

(b) For pan B, resolving vertically:

\[W = T \Rightarrow W = 30 \text{N}\]

This means that the mass of the pan is \(30/g = 3 \text{Kg}\) (\(g\) is the acceleration due to gravity)
(c) With reference to the above diagram, \( W \), the weight of pan and particle is now 50N since the total mass of the pan changed from 3Kg to (3+2)Kg i.e. 5Kg.

Applying Newton’s 2\textsuperscript{nd} law of motion to \( A \):

\[
F = ma
\]

\[
T - 50\sin\alpha = 5f
\]

\[
T = (5f + 30) \text{N}
\]

(d) Applying Newton’s 2\textsuperscript{nd} law of motion to \( B \):

\[
F = ma
\]

\[
50 - T = 5f
\]

\[
T = (50 - 5f) \text{N}
\]

Equating the tensions obtained in parts (c) and (d),

\[
5f + 30 = 50 - 5f
\]

\[
f = 2 \text{ms}^{-2}
\]

---

6.

![Diagram](image)

(a) \( T = \lambda \frac{x}{l} \)

For upper string \( AB \)

\[
x = 1.8 - 1 = 0.8 \text{m}
\]

\[
T_1 = \lambda \frac{0.8}{1} = \frac{4\lambda}{5} \text{N}
\]

For lower string \( BC \)
(b) Particle B is in equilibrium. Thus resolving vertically:

\[ T_1 = T_2 + 3 \]

\[ \frac{4\lambda}{5} = \frac{\lambda}{5} + 3 \]

\[ \frac{3\lambda}{5} = 3 \Rightarrow \lambda = 5 \text{ N} \]

(c) Applying Newton’s 2nd law to the particle with mass of B as 0.3Kg (weight of 30N)

\[ F = ma \]

\[ T_1 - 3 = 0.3a \]

But \( T_1 = \frac{4\lambda}{5} = 4 \text{ N} \)

\[ 4 - 3 = 0.3a \Rightarrow a = 3.33 \text{ ms}^{-2} \]

7. (a)

(b) During the acceleration:

\[ v = u + at \]

\[ 8 = 4 + a(2) \]

\[ a = 2 \text{ ms}^{-2} \]

During deceleration:
\[ v = u + at \]
\[ 0 = 8 + a(2) \]
\[ a = \frac{-4 \text{ m/s}^2}{2} \]

(c) Distance travelled = area under graph

\[ = A_1 + A_2 \]
\[ = \frac{1}{2} (4 + 8)(2) + \frac{1}{2} (4 + 2)(8) \]
\[ = 12 + 24 = 36 \text{ m} \]

(d) Distance travelled by B in 6 seconds is given by,

\[ s = ut \text{ (uniform speed, no acceleration)} \]
\[ s = 6 \times 6 = 36 \text{ m} \]

So after 6 seconds the 2 particles will have travelled the same distance from the same point O. Hence they will be alongside each other.

(e) Particle A:

Distance travelled during acceleration = \( A_1 = 12 \text{ m} \)

Distance travelled at uniform speed = \((8)(2) = 16 \text{ m}\)

Distance travelled during 1 second of deceleration:

\[ s = ut + \frac{1}{2}at^2 \]
\[ s = (8)(1) + \frac{1}{2}(-4)(1^2) \]
Thus the total distance travelled in 5s by particle $A$ is $12+16+6 = 34$ m.

Particle $B$:
Distance travelled in the first 5 seconds = $(6)(5) = 30$ m.
Distance between $A$ and $B = 34 - 30 = 4$ m.