Answer all questions. There are 8 questions in all.

The marks carried by each question are shown.

The total number of marks for all questions in the paper is 100.

Graphical calculators are NOT allowed. Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.
1. (a) Simplify the expression \( \frac{1}{4 + \sqrt{8}} + \frac{3}{4 - \sqrt{8}} \). Give your answer in the form \( a + b\sqrt{8} \), where \( a \) and \( b \) are rational numbers. (Show all your working.)

(b) Find the values of the constants \( A \), \( B \) and \( C \) such that
\[
\frac{18}{x^3 - 9x} \equiv \frac{A}{x} + \frac{B}{x - 3} + \frac{C}{x + 3}
\]
[6, 7 marks]

2. (a) If \( a = x^\frac{1}{2} + x^{-\frac{1}{2}} \) and \( b = x^\frac{1}{2} - x^{-\frac{1}{2}} \), expand \( (ab)^2 + 2 \).

(b) One of the factors of the cubic expression \( x^3 + 2x^2 + 5x + p \) is \( (x + 3) \).
   (i) Find the value of the constant \( p \).
   (ii) Find, by performing algebraic long division, the quadratic factor.
[6, 4, 6 marks]

3. Simplify as much as possible, each of the following:
   (a) \( \log x^3 - \log xy \),  
   (b) \( 3\log x + 2\log \left( \frac{1}{x} \right) \),  
   (c) \( \frac{\log x^4 - \log x^2}{\log x^5 - \log x^3} \).
[1, 3, 3 marks]

4. (a) Consider the quadratic equation \((x - 1)(x + k) = -4\). Find the possible values of the constant \( k \), if this equation has
   (i) equal roots,
   (ii) real different roots.

(b) If one root of the equation \( 64x^2 + ax - 27 = 0 \) is the square of the other, find the value of the constant \( a \).
[8, 3, 7 marks]

5. (a) State, with reasons, which of the following arrow diagrams show function(s):

![Diagram](image)

(b) The function \( f \) is defined by \( f(x) \equiv x^2 + 1 \), \( x \in \mathbb{R} \). Write down the range of \( f \).
[2, 2 marks]
6. The above diagram shows a trapezium ABCD. The coordinates of its vertices are A (–2, –1), B (6, 3), C (3, 5) and D (–1, 3).

(i) Find the length of AD.
(ii) Show that the lines AB and DC are parallel.
(iii) The diagonals of the trapezium meet at E. Find the coordinates of E.
(iv) Show that neither diagonal of the trapezium bisects the other.

[2, 3, 7, 4 marks]

7. (a) If \( \sin^2 \theta = \frac{8}{9} \), where \( \theta \) is obtuse, calculate, without using a calculator, the value of \( \cos \theta \).

(b) Solve the equation \( 4 \sec^2 \theta = 3 \tan \theta + 5 \), for values of \( \theta \) in the range \( 0 \leq \theta \leq 2\pi \).

[4, 9 marks]

8. Differentiate the following with respect to \( x \) and simplify your answers:

(a) \( y = (5 + 2 \ln x)^2 \),
(b) \( y = \frac{e^{2x}}{2 - \sin 2x} \),
(c) \( y = x^2(x + 2)^{10} \).

[3, 5, 5 marks]

END OF QUESTIONS