Directions to candidates

Answer all questions. There are 8 questions in all.

The marks carried by each question are shown.

The total number of marks for all questions in the paper is 70.

Graphical calculators are NOT allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.
1. (a) Rationalise each of the following:
   (i) \( \frac{1}{1+\sqrt{2}} \)
   (ii) \( \frac{1}{\sqrt{2} + \sqrt{3}} \)
   (iii) \( \frac{1}{\sqrt{3} + 2} \)

   Hence, find the sum of \( \frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + 2} \). [5 marks]

   (b) Resolve \( \frac{2x-1}{(x+1)(x^2 + 2)} \) in partial fractions. [5 marks]

2. The vertices of a quadrilateral ABCD are A (6, 7), B (0, 5), C (2, 1) and D (12, 2).

   (a) Find the mid-points of each of the four sides.
   (b) A new quadrilateral is formed by joining the four mid-points.
       Find the gradient of each side of the new quadrilateral.
   (c) Show that this new quadrilateral is a parallelogram. [3, 4, 1 marks]

3. (a) Find the possible values of the constant \( k \), if the quadratic equation \( x^2 + k^2 = (k+1)x \) has equal roots. [4 marks]

   (b) If \( \alpha \) and \( \beta \) are the roots of the quadratic equation \( x^2 - 3x + 5 = 0 \), find the quadratic equation with integer coefficients, whose roots are \( \alpha^2 \) and \( \beta^2 \), without solving the given equation. [5 marks]

4. (a) (i) Show that one of the factors of \( f(x) = x^3 - 2x^2 - 5x + 6 \) is \( x - 1 \).
   (ii) Hence or otherwise, express \( f(x) \) as the product of three linear factors.
   (iii) Factorize \( g(x) = x^3 - 3x^2 - x + 3 \).
   (iv) Hence, simplify as much as possible \( \frac{f(x)}{g(x)} \). [1, 3, 4, 1 marks]

   (b) The magnitude of an earthquake, \( M \), is given by \( M = \log I \), where \( I \) is the intensity of the earthquake.
   (i) What is the intensity of an earthquake whose magnitude is 7.3?
   (ii) Find the magnitude of another earthquake whose intensity is 4 times that of part (i). [2, 2 marks]
5. The function \( f \) is defined by \( f(x) \equiv x^2 + 8x + 23 \), where \( x \in \mathbb{R} \).

(a) Find the values of the constants \( a \) and \( b \) if \( f(x) \equiv (x + a)^2 + b \).

(b) Hence or otherwise, sketch the curve \( y = f(x) \), showing any point where it cuts the axes.

(c) Hence, find the range of \( f \). \[3, 3, 1 \text{ marks}\]

6. Solve each of the following inequalities:

(a) \( 3x - 1 \leq x + 1 \).

(b) \( \frac{4 - x}{x} > -2 \). \[2, 4 \text{ marks}\]

7. (a) Solve the equation \( 6\cos^2\theta - \cos\theta = 1 \), for \( 0^0 \leq \theta \leq 360^0 \). \[4 \text{ marks}\]

(b) By using the identity \( \tan^2\theta + 1 \equiv \sec^2\theta \) or otherwise, prove the identity:
\[ \sec^4\theta - \sec^2\theta \equiv \tan^4\theta + \tan^2\theta. \] \[2 \text{ marks}\]

8. (a) Find the equation of the tangent to the curve \( y = x^3 + 4x^2 + 3x - 1 \) at \( x = 1 \). \[4 \text{ marks}\]

(b) Differentiate each of the following with respect to \( x \), simplifying your answers:

(i) \( y = x^3(3 + 2x)^{12} \) \hspace{1cm} (ii) \( y = \frac{e^x}{\sin 2x} \) \[4, 3 \text{ marks}\]