Directions to candidates

The marks carried by each question are shown.

The total number of marks for all questions in the paper is 100.

Graphical calculators are NOT allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

Where necessary take $g=10\text{ms}^{-2}$
1. **ABCD** is a parallelogram. Vector \( AB = 2x \) and vector \( AD = 2y \). Referred to an origin \( O \), the position vector of \( A \) is given by \( OA = a \). *(See diagram)*

(a) Show that the position vector of \( B \) is \( OB = (a + 2x) \) and find similar expressions in terms of \( a, x \) and \( y \), as required, for the position vectors of \( D \) and \( C \).

\( G \) is the point with position vector \( OG = (a + x + y) \) and \( Z \) is a general point with position vector \( OZ = z \).

(b) Show that \( DG = GB \) and \( AG = GC \) and hence prove that \( G \) is the point of intersection of diagonals \( BD \) and \( AC \).

(c) Prove that:

\[
ZA + ZB + ZC + ZD = 4ZG.
\]

In terms of the unit perpendicular vectors \( i \) and \( j \), the position vectors of the points \( P, Q, R \) and \( S \), with respect to \( O \), are respectively \((i), (4i + 2j), (3i + 4j)\) and \((2j)\).

(d) Prove that the quadrilateral \( PQRS \) is a parallelogram, by showing that a pair of its opposite sides are equal in length and parallel. Give the lengths of the sides of \( PQRS \).

(e) Determine the position vector of the point where the diagonals \( PR \) and \( QS \) meet.

\[2, 4, 3, 4, 2 \text{ marks}\]

---

2. A small ring \( A \), of weight 10 N, is slung on a fixed, smooth, vertical rod. The ring is connected by a stretched, light, elastic string to a particle \( B \), of weight 70 N, which lies on the rough, horizontal surface of a table. The elastic string passes over a smooth, fixed pulley \( C \). When the system is in equilibrium \( A \) and \( B \) are on one horizontal line, 120 cm apart. \( C \) lies 25 cm vertically above this line, equidistant from \( A \) and \( B \). *(See diagram)*

(a) Show that the sections \( AC \) and \( BC \) of the string make equal angles \( \sin^{-1}(5/13) \) with the horizontal line \( AB \). Find the exact value of the cosine of these angles.

(b) Draw two separate diagrams showing the directions of the forces on \( A \) and on \( B \).

(c) By considering the equilibrium of ring \( A \), show that the common tension in the string is 26 N.

(d) Determine the frictional force and the normal reaction acting on the particle \( B \).

(e) Given that the natural length of the elastic string is 78 cm, find its modulus of elasticity.

\[1, 4, 2, 4, 3 \text{ marks}\]
3. A rigid body is in equilibrium under the action of three forces, \( P \), \( Q \) and \( R \). The lines of action of forces \( P \) and \( Q \) intersect at the point \( O \).

(a) Show, by taking moments about \( O \), that the line of action of force \( R \) also passes through the point \( O \).

\( ABCD \) is a uniform square lamina of side 30 cm and weight 56 N. It is kept in equilibrium, with side \( AB \) horizontal and uppermost, by an inextensible, light string which has one end attached to the lamina at the midpoint, \( E \), of \( AB \) and the other end attached to a fixed point \( F \), 20 cm vertically above \( A \). The lamina is in \textit{rough} contact with a peg at the point \( G \) on its side \( AC \), where the distance \( AG \) is 8 cm. (See diagram)

(b) Find the distance \( GE \) and give the length of the string \( FE \).

(c) By considering the normal and the frictional reactions of the peg on the lamina as a \textit{single} resultant reaction, explain carefully why triangle \( FGE \) has its sides parallel to the three forces keeping the lamina in equilibrium. (That is triangle \( FGE \) is a Triangle of Forces.)

(d) Find the magnitude of the resultant reaction at \( G \) and the tension in the string \( EF \).

(e) Determine the frictional force on the lamina \( ABCD \). \( [3, 1, 5, 3, 2 \text{ marks}] \)

4. \( AB \) is a uniform, horizontal rod of length 200 cm and weight 20 N. The rod, which has a small load of weight 10 N attached at a point \( C \), distant \( x \) cm from \( B \), is in equilibrium with its ends in \textit{smooth} contact with two perpendicular planes. The plane supporting the end \( A \) makes an angle \( \theta = \tan^{-1}(4/3) \) with the horizontal. The vertical plane containing the rod \( AB \) cuts the two perpendicular planes at right angles. (See diagram)

(a) Draw a diagram showing the forces acting on the system of rod plus particle.

(b) By resolving forces in a certain direction, or otherwise, show that reaction on the rod at \( A \) is 18 N.

(c) Determine the magnitude of the reaction on the rod at \( B \).

(d) By taking moments about a suitable point find the value of the distance \( x \). \( [2, 3, 3, 4 \text{ marks}] \)
5. \(ABC\) is an equilateral triangle with sides 4 m long. \(AO\) is the perpendicular from \(A\) to \(BC\). The perpendicular distances of \(O\) from the lines \(AB\) and \(AC\) are both equal to \(d\). (See diagram (a))

\[
\begin{align*}
\text{(a)} & \quad \text{Show that the exact length of } AO, \text{ as well as that of the perpendiculars from } B \text{ to } AC \text{ and from } C \text{ to } AB, \text{ is } (2\sqrt{3}) \text{ m. Determine the exact value of } d. \\
\text{(b)} & \quad \text{Forces of magnitude } 10 \text{ N, } P, Q \text{ and } R \text{ act along } AB, BC, CA \text{ and } OA \text{ respectively in the directions indicated by the order of the letters. (See diagram (b))}
\end{align*}
\]

5. \(ABC\) is an equilateral triangle with sides 4 m long. \(AO\) is the perpendicular from \(A\) to \(BC\). The perpendicular distances of \(O\) from the lines \(AB\) and \(AC\) are both equal to \(d\). (See diagram (a))

\[
\begin{align*}
\text{(a)} & \quad \text{Show that the exact length of } AO, \text{ as well as that of the perpendiculars from } B \text{ to } AC \text{ and from } C \text{ to } AB, \text{ is } (2\sqrt{3}) \text{ m. Determine the exact value of } d. \\
\text{(b)} & \quad \text{Forces of magnitude } 10 \text{ N, } P, Q \text{ and } R \text{ act along } AB, BC, CA \text{ and } OA \text{ respectively in the directions indicated by the order of the letters. (See diagram (b))}
\end{align*}
\]

6. A train starts from rest at station \(A\) and comes to rest, six minutes later, at station \(B\), after travelling a total distance of 5,400 metres. The train is uniformly accelerated for the first two minutes of its motion, when it reaches its maximum speed \(v\) metres per second, and then runs for another three minutes at this maximum speed. In the last minute of its journey it is uniformly retarded to rest at station \(B\).

\[
\begin{align*}
\text{(a)} & \quad \text{Draw the velocity (m s}^{-1}) \text{ – time (second) graph for the motion of the train, labelling the significant data given.} \\
\text{(b)} & \quad \text{Find an expression, in terms of } v, \text{ for the total area under the graph and hence deduce that } v = 20. \\
\text{(c)} & \quad \text{Determine the acceleration of the train in the first two minutes of its motion and show that its retardation in its last minute is } (1/3) \text{ metre per second squared.} 
\end{align*}
\]

On another journey, along the same track to station \(B\), the train passes through station \(A\) with its maximum speed of 20 metres per second and then decelerates, as before, at the rate of \((1/3)\) metre per second squared to come to rest at \(B\).

\[
\begin{align*}
\text{(d)} & \quad \text{Draw the velocity-time graph of the train in this other journey and deduce the time it takes to travel from station } A \text{ to station } B \text{ in this case.} \\
& \quad \text{[3, 4, 3, 5 marks]} 
\end{align*}
\]
7. Two particles $A$ and $B$, of masses 1 kg and 2 kg respectively, are connected by a light, inextensible string. The taut string lies along a line of greatest slope of a smooth plane, which is 0.8 m long and inclined at 30° to the horizontal, and then passes over a smooth pulley $P$ at the top of the plane. Initially $A$ lies at rest at the foot of the plane and $B$ is supported just below the pulley $P$ and 0.4 m above the ground. (See diagram) The system is then let go.

(a) Draw a diagram showing both the directions of the forces acting on $A$ and its acceleration while both particles are still in motion. Draw a similar diagram for $B$.
(b) Apply Newton’s Second Law of Motion to each of the particles $A$ and $B$, while both are still in motion. Hence find the common acceleration of $A$ and $B$.
(c) Show that $B$ hits the ground with speed 2 m s$^{-1}$.

When $B$ hits the ground it stops immediately while $A$ continues its motion along the inclined plane with the string slack.

(d) Determine the retardation of $A$, while $B$ is at rest on the ground.
(e) Show that $A$ will come to rest just as it reaches the pulley $P$. [3, 5, 2, 2, 3 marks]

END OF QUESTIONS