Directions to candidates

Answer all questions. There are 7 questions in all.

The marks carried by each question are shown.

The total number of marks for all questions in the paper is 70.

Graphical calculators are NOT allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

Where necessary take $g = 10 \text{ms}^{-2}$
1. \(ABCD\) is a parallelogram. \(E\) is the midpoint of side \(AB\) and \(F\) is a point on \(DC\) such that \(DF = (1/4)DC\). The point \(G\) lies on the diagonal \(AC\) and \(AG = (2/5)AC\). Vector \(\vec{AB} = 20 \times\) and vector \(\vec{AD} = 20 \times\). (See diagram.)

(a) Find expressions, in terms of \(x\) and \(y\) as required, for vectors \(\vec{AE}\) and \(\vec{DF}\). Show that \(\vec{EF} = 20y - 5x\)

(b) Find vectors \(\vec{AC}\) and \(\vec{AG}\) in terms of \(x\) and \(y\). Show that \(\vec{EG} = 8y - 2x\).

(c) Prove that vectors \(\vec{EF}\) and \(\vec{EG}\) are parallel.

Given that \(\vec{AD} = (40i + 40j)\) and \(\vec{EF} = (20i + 40j)\), where \(i\) and \(j\) are unit perpendicular vectors,

(d) find vectors \(x\) and \(y\) in terms of \(i\) and \(j\);

(e) show that \(\vec{EG} = 8(5)^{1/2}\). 

[3, 2, 2, 2, 2 marks]

2. One end of a light, inextensible string is fastened to a smooth ring \(A\), of weight \(W\), which is slung on a fixed vertical rod \(XY\). The string passes over a smooth, fixed pulley \(P\) and has a particle \(B\), of weight 20 N, attached at its other end. The distance between \(P\) and the upper end \(X\) of the rod is 1.2 m and the line \(PX\) is horizontal. In equilibrium \(B\) hangs vertically below \(P\), and \(A\) lies at rest 1.6 m below \(X\). (See diagram (a) below)

(a) Show that if \(\theta\) is the acute angle the string makes with the upward vertical at \(A\), then \(\sin \theta = 3/5\) and find the value of \(\cos \theta\).

(b) Draw diagrams of the forces acting on \(A\) and \(B\). By considering the equilibrium first of \(B\) and then of \(A\) find the tension in the string and show that \(W = 16\) N.

When the smooth ring \(A\) is replaced by a similar rough ring \(C\) of the same weight, \(C\) is in limiting equilibrium 0.9 m below \(X\). (See diagram (b) above)

(c) By drawing suitable force diagrams, find the normal reaction and frictional force exerted by the rod on the ring \(C\) and determine the coefficient of friction between the rod and the ring. 

[1, 3, 6 marks]
3. Three coplanar forces $P$, $Q$ and $R$ keep a rigid body in equilibrium. The lines of action of $P$ and $Q$ intersect at the point $O$.

(a) Prove, by taking moments about $O$ or otherwise, that the line of action of the third force $R$ must also pass through $O$.

$ABCD$ is a uniform rectangular lamina with its sides $AB$ and $AD$ being 260 cm and 100 cm long respectively and whose weight, considered to act at its centre, is 195 N. The lamina has its vertex $A$ smoothly hinged to a fixed point about which it can turn freely in a vertical plane. A taut, light, elastic string attached to $B$, and parallel to $BC$, keeps the lamina at rest with $B$ uppermost and the diagonal $BD$ vertical. (See diagram) The string is 120 cm long when unstretched, and has length 160 cm in the equilibrium position.

(b) Show that triangle $BDA$ has its sides parallel to the 3 forces which keep the lamina in equilibrium and is therefore a Triangle of Forces. Hence, or otherwise, determine the tension in the string and the reaction on the lamina at $A$.

(c) Find the modulus of elasticity of the elastic string. [2, 6, 1 marks]
4. A uniform rod $AB$, of mass 20 kg and length 150 cm, is inclined at an angle $\theta$ to the horizontal, where $\tan \theta = \frac{3}{4}$, and has its lower end $A$ in contact with a rough horizontal plane. The rod is kept in equilibrium by means of two forces, $P$ and $2P$, acting at its upper end $B$. The force $P$ acts vertically upwards while $2P$ is directed horizontally towards $A$. (See diagram)

(a) By considering all the forces acting on the rod $AB$ and taking moments about $A$, or otherwise, show that $P = 40$ N.

(b) Find the normal reaction and frictional force on the rod and show that their resultant is perpendicular to the resultant force acting at $B$. [4, 4 marks]

5. $ABCD$ is a square of side 1 m. A coplanar system of forces consists of five forces having magnitudes 2 N, 4 N, 4 N, 2 N, $2(2)^{1/2}$ N and acting along $AB$, $BC$, $CD$, $DA$ and $AC$ respectively in the directions indicated by the order of the letters. (See diagram (a))

(a) Determine the sum of the moments of the 5 forces about $A$ and hence show that the system cannot be in equilibrium.

(b) Find the sum of the components of the 5 forces in the direction of the vector $AD$ and hence show that the system cannot resolve to a couple.

The resultant of the system is a force acting at $E$, on $AB$ produced and distant $d$ from $A$, with components $X$, $Y$ in the directions $AB$ and $AD$ respectively. (See diagram (b))

(c) Find the values of $X$, $Y$ and $d$.

(d) Find the moment of the couple which must be added to the original system of 5 forces if the new resultant is a force acting at $B$.

(e) Find the magnitude of the force acting at $A$ in the direction $AD$ which must be added to the original system of 5 forces if the new resultant is a force acting at $B$. [2, 2, 4, 2, 1 marks]
6. At time $T = 0$ minutes, two trains, $P$ and $Q$, moving along straight parallel tracks, both start from rest at station $A$. The trains then come to rest together again at station $B$, 6 km away, at $T = 12$ minutes.

(a) Find the average speed in kilometres per hour of the two trains.

Train $P$, starting at $A$, moves with constant acceleration $f_1$ metre minute$^{-2}$ in the first 4 minutes, constant speed $u_1$ metre minute$^{-1}$ for the next 4 minutes and constant deceleration $f_1$ metre minute$^{-2}$ in the last 4 minutes of its motion.

Train $Q$ leaves $A$ moving with constant acceleration $f_2$ metre minute$^{-2}$ until it reaches its top speed $u_2$ metre minute$^{-1}$ after 6 minutes and then decelerates at the constant rate of $f_2$ metre minute$^{-2}$ for the next 6 minutes.

(b) Draw, on the same axes, the velocity-time graphs of the motions of the two trains from $A$ to $B$.

(c) Show that $u_1 = 750$ and determine the values of $f_1$, $u_2$ and $f_2$.

(d) Find the two values of $T$ when the trains are moving with the same velocity.

(e) Determine the value of $T$ when train $Q$ is most ahead of train $P$.

[1, 2, 5, 2, 1 marks]

7. Two particles, $A$ and $B$, of mass 9 and 6 respectively, are attached at either end of a long, light, inextensible string which slides over a smooth, fixed pulley attached to a fixed point high above the ground. To the particle $B$, of mass 6, is glued another particle, $C$, of mass 5.

Initially the three particles are held at the same horizontal level vertically beneath the pulley with the string joining them taut. When the system is let go the three particles start moving from rest with $B$ and $C$ moving as one object.

(a) By applying Newton’s Second Law of motion to $A$ separately, and to $B$ and $C$ together, considered as one entity, determine the tension in the string and show that the common acceleration of the three particles is 1 m s$^{-2}$.

(b) Find the velocity of the particles after 2 seconds and show that at that time $A$ is at a vertical height of 4 m above $B$ and $C$.

After the three particles had been in motion for 2 seconds, the particle $C$ becomes unstuck from its connection with $B$ and falls while $B$ remains attached to the string.

(c) Determine the common acceleration of the particles $A$ and $B$ in this new motion.

(d) Find the distance between the vertical heights of $A$ and $B$ at the instant when they are both momentarily at rest.

[3, 2, 3, 2 marks]

END OF TEST