1. (a) $OA = |OA| = |(-15i - 20j)| = 5\sqrt{31 - 4} \cdot j = 5\{(-3)^2 + (-4)^2\}^{1/2} = 25; \ OB = |25i| = 25; \ OC = |7i + 24j| = \sqrt{(-7)^2 + 24^2}^{1/2} = 25. \ \text{Hence the lengths of } OA, \ OB, \ OC \ \text{are equal.}

(b) LHS = 5\{(AO + OB) + (AO + OC)\} = 5\{2(AO)(OB) + (OC)\} = 5\{2(15i + 20j) + (25i) + (-7i + 24j)\} = 5\{30 + 25 - 7i\} + (40 + 24j)\} = 240i + 320j = 16(15i + 20j) = \text{RHS.}

(c) $OD = OC + \frac{1}{2} CB = OC + \frac{1}{2} (CO + OB) = \frac{1}{2} (OC + OB) = \frac{1}{2} \{25 - 7i + 24j\} = 9i + 12j = (3/5)(15i + 20j) = (3/5)AO$. Since $OD = (3/5)AO$, then $OD$ and $AO$ have the same direction and $A, O, D$ are collinear.

(d) Given implies $F_{OA} = 100N\{(1/2)OA\} = 100N\{(1/25)(-15i - 20j)\} = 4(-15i - 20j)N$. Similarly $F_{OB} = 4(25i)N$ and $F_{OC} = 4(-7i + 24j)N$.

(e) Vector resultant of forces is $F_{OA} + F_{OB} + F_{OC} = 4\{(-15 + 25)j + (-20 + 24)j\}N = 4\{3i + 4j\}N$. Since this resultant acts at $O$ and its direction, that is $(3i + 4j)$, is parallel to $OD = 3\{3i + 4j\}N$. Hence the lengths of forces $AO, OB, OC$ are equal.

2. (a) Since triangle $AOB$ is isosceles then the string makes an angle of $\{(180^0 - 120^0)/2\} = 30^0$ with each rod. Also, $AB = \{AO^2 + OB^2 - 2(AO)(OB) \cos 120^0\}^{1/2} = \{1^2 + 1^2 - 2(1)(1)(-1/2)\}^{1/2} = 3^{1/2} \text{ m.}

(b) Since the system is in limiting equilibrium, with $B$ tending to slide down the rod while $A$ tending to slide towards $O$, the frictional force on ring $A$ is maximum and the forces are directed as shown.

3. (a) Since the lamina $ABCD$ is uniform and $AB$ is horizontal, its Centre of Mass is vertically below $O$. Hence the line of action of the weight passes through $O$ and is parallel to $CB$ which is vertical.

Since $\tan(OCB) = 60/32 = 15/8$, then the direction of the normal reaction of the plane on the lamina, $N_C$ which acts at $C$, is along $CO$.

Now since the lines of action of the weight and $N_C$ meet at $O$, the line of action of the third force which keeps the lamina in equilibrium, that is the reaction $N_A$ of the hinge, must also pass through $O$ by the Three-Force Result. Thus, $N_A$ is parallel to the side $OB$ of triangle $COB$. Hence the three forces are parallel to the sides of triangle $COB$. 

Diagram: 

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**SOLUTION OUTLINES**

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(b) From the above, since triangle $COB$ has its sides parallel to the 3 forces in equilibrium it must be a Triangle of forces. Hence we have that the magnitudes of the 3 forces are proportional to the lengths of the sides to which they are parallel. Hence

$$BC/64N = CO/N_C = OB/N_A$$

or $32cm/64N = 68cm/N_C = 60cm/N_A$ implying $N_C = 136N$, $N_A = 120N$.

(c) By the Three-Force Result, when the lamina is maintained in equilibrium by the three forces: the weight, the vertical tension of the string acting at $B$, and the new normal reaction of the hinge at $A$, since the first two forces are parallel then so must be the third, that is the new reaction at $A$, $R_A$. Hence $R_A$ is vertical.

By the Principle of Moments, the sum of the sum moments of this system about any coplanar point must be equal to the moment of their resultant. This sum must be zero since the system is in equilibrium. Hence taking moments about $B$, anticlockwise moment of weight + moment of $R_C$ add up to zero. This means that the moment of $R_A$ must be clockwise, implying $R_A$ is directed vertically upwards, and has magnitude $64N(60cm)/(120cm) = 32N$.

4. (a) The external forces acting on the composite system of rod plus ring are as follows, where $X$ and $Y$ are the horizontal and vertical components of the reaction of the hinge at $A$ on the rod.

(b) Taking moments about $A$, by the Principle of Moments, we have:

$$T(80 \sin 30^\circ \text{ cm}) = T(40 \text{ cm}) = 120N(60 \text{ cm}) + 80N(x \text{ cm})$$

or $T = (180 + 2x) N$

(c) Equilibrium of system implies on resolving horizontal and vertically respectively:

$$X = T \cos 30^\circ = (90 + x)3^{1/2} N,$$

$$Y = 120 + 80 - T \sin 30^\circ = (200 - 90 - x) N$$
or $(110 - x) N$.

(d) When the resultant reaction is directed horizontally $Y = 0$, so that $x = 110 \text{ cm}$. In this case the resultant reaction is $X = (90 + 110)3^{1/2} N$ or $200(3^{1/2}) N$.

5. (a) The components of the 30N in directions $AB$, $DA$ respectively are: $30N(4/5) = 24N$, and $30N(3/5) = 18N$

The components of $P$ in directions $AB$, $DA$ respectively are: $(-3P/5) N$, and $(4P/5) N$

Since the moments of $X$ and $Y$ about $C$ are both zero, as the forces act at $C$, the sum of moments of the system about $C$ is equal to the moment of the 30N plus the moment of $P$ about $C$. By the Principle of Moments this is equal to the sum of the components of these forces.

This clockwise sum of moments is: $(24N)(10cm) + (18N)(20cm) + (-3P/5)(10cm) = (600 - 6P) N \text{ cm}$.

(b) In general for any coplanar system of forces acting on a rigid body we have:

$$\sum \mathbf{F}_{\text{system}} = \mathbf{F}_{\text{resultant}}$$

and $$\sum \text{ moments about } O_{\text{system}} = \sum \text{ moments about } O_{\text{resultant}}$$

(i) So when the system is in equilibrium,

- taking moments about $C$ : $600 - P = 0$ or $P = 100 N$
- resolving in direction $AD$ : $Y - (4P/5) - 18 = 0$ or $Y = 98 N$
- resolving in direction $AB$ : $X + 24 - (3P/5) = 0$ or $X = 84 N$
6. (a), (c) The velocity-time graphs of the motion of cars $P$ and $Q$ (in grey) are as shown. In both cases we have

area under graph = distance and gradient = acceleration

(b) Distance travelled by car $A$ in last 5s = area under graph of $A$’s motion = $\{(16 \text{ ms}^{-1})(2s +5s)/2\} = 56\text{m}$. Retardation = value of negative gradient of graph of $A$’s motion in last 3s = $(16 \text{ ms}^{-1})/(3s) = 5.33 \text{ ms}^{-2}$.

d) Distance travelled by car $B$ in last 5s = area under graph of $B$’s motion = $116\text{m} – 56\text{m} = 60\text{m}$

Hence $\{(20 \text{ ms}^{-1})(7s +5s)/2\} = 10T +50 = 60$; implying $T = s$, and retardation of $B = 20/(5-1) = 5 \text{ ms}^{-2}$

e) The time when the cars will have the same speed apart from at the end corresponds to the time where the graphs cross each other somewhere between $T = 1s$ and $2s$. Let this time be $t$.

Considering the motion of $B$ between 1s and time $t$, retardation = $(20-16)/(t-1) = 5$. This implies $t = 1.80s$.

7. (a) The Force-Acceleration diagrams when both blocks are in motion are as follows.

(b) $F = ma$ on $A$ along the plane gives: $60 \sin 30^\circ – T = 6f$ or $30 – T = 6f$.

c) $F = ma$ on $B$ vertically: $N_B = 40\text{N}$; horizontally: $T – (N_B/4) = 4f$ or $T – 10 = 4f$.

From the above equations: $(30 – T) + (T – 10) = (6f) + (4f)$ giving $f = 2 \text{ ms}^{-2}$.

d) From the above since the acceleration is constant we may use $v^2 = u^2 + 2as$, for the motion when both blocks are in motion. This gives $v^2 = (0^2) + 2(2 \text{ ms}^{-2})(1\text{m})$ or $v$, the velocity with which $B$ ends its first motion and starts its second motion while $A$ is at rest, is $2 \text{ ms}^{-1}$.

In this second motion $T$ becomes zero so that the only horizontal force on $B$ is the frictional $N_B/4$ or 10N. Since $B$’s mass is 4kg this force gives it a retardation of $(10\text{N})(4\text{kg})/(5/2)\text{ms}^2$ by Newton’s 2nd Law.

Hence applying again $v^2 = u^2 + 2as$ for the entire second motion we get: $v^2 = 2^2 + 2(-5/2)(s)$ or $s = 0.8m$.

This means that the total distance before $B$ comes to rest is $(1 + 0.8)m = 1.8m$ meaning it will just reach $P$. 