Directions to candidates

The marks carried by each question are shown.
The total number of marks for all questions in the paper is 70.
Graphical calculators are NOT allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.
Where necessary take \( g = 10 \text{ms}^{-2} \)
1. \(ABC\) is a triangle. The position vectors of the vertices \(A, B\) and \(C\) referred to an origin \(O\) are \(OA = a, \ OB = b\) and \(OC = c\) respectively. The point \(X\), with position vector \(x\), is on the side \(AB\) and divides \(AB\) in the ratio \(2:1\) such that \(AX = 2XB\). The point \(Y\) is the midpoint of \(AC\) and has position vector \(y\). (See diagram)

(a) Show that \(x = \frac{1}{3}a + \frac{2}{3}b\), and similarly find \(y\) in terms of \(a\) and \(c\).

In terms of the unit, perpendicular vectors \(i\) and \(j\), the position vectors of vertices \(A, B\) and \(C\) are: \(a = (12i + 9j), \ b = (-6i)\) and \(c = (-9j)\).

(b) Find the position vectors of \(X\) and \(Y\) in terms of \(i\) and \(j\). Hence, or otherwise, prove that \(X\) is collinear with \(O\) and \(C\). Similarly, prove that \(YOB\) is one line.

A force \(P_B\) has magnitude 400N and acts at \(B\) in the direction of the vector \(BY\). Another force \(Q_C\) has magnitude 300N and acts at \(C\) in the direction of \(CX\).

(d) Find the magnitude of the resultant of the two forces \(P_B\) and \(Q_C\).

(e) Explain why the line of action of this resultant passes through both points \(O\) and \(A\).

[3, 1, 2, 2, 3 marks]

2. One end of a light, elastic string, of natural length 30cm and modulus of elasticity 75N, is attached to a fixed point \(O\). The other end is tied to a ring, of mass 2kg, slung on a horizontal, fixed rail and resting at point \(A\), 48cm vertically beneath \(O\). (See diagram (a))

(a) Apply Hooke’s law to determine the tension in the elastic string.

(b) Explain why the frictional force on the ring is zero. With the help of a suitable force diagram find the magnitude and direction of the reaction of the rail on the ring.

The ring is now moved laterally 14cm on the rail from its position \(A\) to its new position of rest \(B\) such that now the string makes angle \(\theta\) with the vertical. (See diagram (b))

(c) Show that \(\sin\theta = \frac{7}{25}\) and find a similar expression for \(\cos\theta\).

(d) Determine the new tension in the elastic string.

(e) Draw a diagram showing all the forces acting on the ring at \(B\) and hence find the normal reaction and frictional force on the ring.

[2, 2, 1, 2, 3 marks]
3. A rigid body is in equilibrium under the action of a number of forces.

(a) What can you say about the lines of action of the forces if:
   (i) only 2 forces act on the body;  (ii) 3 non-parallel forces act on the body.

A non-uniform square lamina $ABCD$, of side 100cm and weight 20N, is held at rest in a vertical plane with its side $AB$ uppermost and horizontal by means of two light strings, coplanar with $ABCD$ and attached at $A$ and $B$. The string at $A$ makes angle $\tan^{-1}(4/3)$ with the vector $BA$, while that at $B$ makes angle $\tan^{-1}(3/4)$ with $AB$. (See diagram)

(b) By resolving forces, or otherwise, find the tension in each of the two strings.
(c) Show that the centre of mass of the lamina lies 36cm away from the side $AD$.

The string at $B$ is removed and the lamina is now at rest suspended only by the string at $A$. In this case the sides $AB$ and $AD$ both make an angle of $45^0$ with the horizontal.

(d) Explain why the string must be vertical and the centre of mass of the lamina must lie vertically below $A$.
(e) Deduce the distance of the centre of mass of the lamina from the side $AB$.

[2, 3, 2, 2, 1 marks]

4. A uniform rod $AB$, of weight 120N and length 80cm, has a ring of weight 20N rigidly attached to its end $B$. The rod is freely hinged at a point $C$ of its length, 20 cm from $A$, and is kept in equilibrium, with $A$ uppermost and $AB$ making $30^0$ with the downward vertical, by means of light inextensible string tied to the ring at $B$ and making $90^0$ with $AB$. The string lies in the same vertical plane of $AB$. (See diagram)

(a) Draw a diagram showing the (external) forces which keep the composite body, consisting of rod plus ring, in equilibrium.
(b) By taking moments about a suitable point show that the tension in the string is 30N.
(c) Find the magnitude and direction of the reaction of the hinge on the rod in newtons and degrees respectively correct to 2 decimal places.

[2, 3, 3 marks]
5. \(ABCD\) is a square of side 2m. \(E\) and \(F\) are the midpoints of the sides \(AD\) and \(BC\) respectively. Five coplanar forces of magnitudes \(P, Q, R, 20N\) and \(20N\) act along \(DA, BA, DC, BC\) and \(EF\) respectively in the direction indicated by the order of the letters. (See diagram)

Find the values of \(P, Q\) and \(R\) in the separate cases when the system of five forces

- (i) is in equilibrium;
- (ii) resolves to a couple of moment 10Nm in the sense \(CBA\).
- (iii) is a single force of magnitude 20N acting along \(AD\) in the direction of \(AD\)

[3, 4, 3 marks]

6. Two trains, \(A\) and \(B\), moving in a straight line, cover the same distance \((S)\) kilometres in the same time \((T)\) hours.

Train \(A\) starts from rest moving with constant acceleration until it reaches its maximum speed \((V_A)\) kilometres per hour after time \((T/3)\) hours. It then continues moving with this maximum speed for a further time \((T/3)\) hours after which it decelerates uniformly to rest in the last \((T/3)\) hours of its motion.

Train \(B\) starts from rest and moves with constant acceleration for a time \((T/2)\) hours and then, having reached its maximum speed \((V_B)\) kilometres per hour, starts decelerating to rest for the remaining \((T/2)\) hours of its trip.

(a) Draw the velocity time graph for the motion of each train clearly labelling the significant data given.
(b) Express the maximum velocity of each train in terms of \(S\) and \(T\) and hence show that \(V_A/V_B = 3/4\).
(c) Express the acceleration of each train in terms of \(S\) and \(T\) and hence show that the ratio of train \(A\)'s acceleration to that of train \(B\)'s is 9/8.
(d) Prove that at time \((3T/8)\) hours after each train has started its journey the two trains would have the same speed.

[3, 3, 2, 2 marks]
7. A small block $A$, of mass $m$, rests in equilibrium on the smooth surface of a plane inclined at an angle $\theta = \tan^{-1}(4/3)$ to the horizontal. A light inextensible string passing over a smooth pulley $P$ at the top of the inclined plane connects block $A$ to another block $B$, of mass 4kg, hanging freely below $P$. (See diagram (a))

![Diagram (a) equilibrium of A and B]

(a) Draw two diagrams showing the directions of the forces acting on $A$ and on $B$.
(b) Considering the equilibrium of the two blocks separately find the value of $m$.

The block $A$ is now removed and a third block, $C$, of mass $M$, is placed in the same position previously occupied by $A$. When the new system is released from rest, $B$ goes up a distance of 1metre in 1 second. (See diagram (b))

(c) Using the equations for constant acceleration show that the common acceleration of $B$ and $C$ is $2\text{ms}^{-2}$.
(d) With the help of appropriate force-acceleration diagrams, apply Newton’s second law of motion to both blocks and hence determine the value of $M$.

[2, 3, 1, 5 marks]