Directions to candidates

Attempt any 7 questions.

The marks carried by each question are shown.

The total number of marks for all questions in the paper is 70.

Graphical calculators are NOT allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

Where necessary take $g=10\text{ms}^{-2}$
ATTEMPT ANY 7 QUESTIONS

1. $ABC$ is an equilateral triangle having vertices $A(0, 0)$, $B(2, 0)$ and $C(1, \sqrt{3})$, all coordinates being in metres. $D$ and $E$ are midpoints of $AB$ and $BC$ respectively. A force of magnitude 2 N acts along $AC$ in the direction $\overrightarrow{AC}$. Forces of magnitude $P$ and $Q$ act along $AB$ and $BC$ respectively. The vectors $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors along the $x$-axis and $y$-axis respectively.

i) Express the three forces in the system in $\mathbf{i}, \mathbf{j}$ notation and write the resultant force in terms of $P$ and $Q$.

ii) Find the values of $P$ and $Q$ and the direction of these forces if the system reduces to a couple. Find also the magnitude and sense of the couple.

iii) Given that the above system reduces to a single force along $DE$, by taking moments or otherwise, find the values of $P$ and $Q$ and the direction of these forces. Hence find the magnitude of the resultant.

[3, 4, 3 marks]

2. A framework $LMN$ consists of three uniform rods $LM$, $MN$, and $NL$, of length 8 cm, 6 cm, and 10 cm respectively. The mass of a 1 cm length of rod is 0.1 kg. The rods are joined at the vertices with pins of mass 0.1 kg each to form a right-angled triangle $LMN$.

i) Find the distance of the centre of mass of the structure from the sides $LM$ and $MN$.

ii) The system is then freely suspended from $L$. Find the angle which the side $LM$ makes with the lower vertical.

iii) Find the horizontal force required to maintain the triangular framework in equilibrium with $LM$ in a vertical position.

[6, 2, 2 marks]
3. A solid block of weight $2W$ rests on a plane inclined at an angle of $30^\circ$ to the horizontal. The coefficient of friction between the block and the plane is $\frac{1}{2}$.

i) Show that unless the block is held it will move downwards, and find its acceleration in terms of $g$.

ii) If a horizontal force $P$ just prevents the block from sliding down the plane, find its magnitude in terms of $W$.

[4, 6 marks]

4. A particle $P$ of mass 2 kg moves under the influence of a single force $\mathbf{F}$. The position vector of $P$ at time $t$ is given by $\mathbf{r} = (3t + 1)\mathbf{i} + (2t^2 - 3t + 1)\mathbf{j}$. Find:

i) the velocity and the acceleration vectors of the particle;

ii) the kinetic energy of the particle at time $t$, and at $t = 2$;

iii) the force $\mathbf{F}$ and the work done by $\mathbf{F}$ in the time interval from $t = 1$ to $t = 2$.

**Hint:** The kinetic energy is given by $\left(\frac{1}{2}m\mathbf{v} \cdot \mathbf{v}\right)$.

[2, 3, 5 marks]

5. A car of mass 1500 kg moves with a constant speed of 6 ms$^{-1}$ up a slope of inclination $\sin^{-1}\frac{1}{6}$. The engine of the car is working at a rate of 18 kW.

i) Find the resistance to motion in newtons.

ii) If the power and the magnitude of the resistance remain unchanged, find the acceleration of the car when it is moving down the slope at 3 ms$^{-1}$.

[5, 5 marks]
6. A projectile is fired from \( O \) on level ground with velocity \((U\mathbf{i} + V\mathbf{j})\) ms\(^{-1}\), where \( \mathbf{i} \) and \( \mathbf{j} \) are the unit vectors along the horizontal and upward vertical directions respectively.

i) Write down the vector expressions for the acceleration, velocity \( \mathbf{v} \) and position \( \mathbf{r} \), relative to \( O \), of the projectile at time \( t \).

ii) Find the Cartesian equation of path of the projectile in terms of \( U, V \) and \( g \).

iii) Find the time of flight and the horizontal range of the projectile.

iv) State the angle which gives maximum range \( OP \) and how it can be interpreted in terms of \( U \) and \( V \). Hence show that the maximum range on level ground is given by \( OP = \frac{2U^2}{g} \).

v) A target is now placed at a height \( h \) above \( P \). Find in terms of \( U, g \) and \( h \) the velocity with which the projectile must be fired if it is to hit this target without changing the angle of projection stated in (iv).

[2, 2, 2, 1, 3 marks]
7. (a) A uniform rod $AB$ of weight $W$ and length $6a$ is kept in limiting equilibrium with $A$ against a vertical wall by means of a light inextensible string, of length $4a$, attached to the mid-point of the rod. The other end of the string is attached to the wall at $C$, a distance $5a$ vertically above $A$. The rod and the string lie in one vertical plane perpendicular to the wall.

   i) Using the “three-force” result state which point is common to the lines of action of the forces involved (considering the total reaction at $A$ instead of the normal reaction and the frictional force individually). Hence state the direction of the total reaction at $A$.

   ii) Find a triangle of forces in the above diagram.

   iii) Using the triangle of forces, or otherwise, find the magnitude of the tension and the total reaction at $A$ in terms of $W$.

   iv) Find also the angle of friction and the coefficient of friction between $A$ and the wall.

(b) Using a similar setting to that above, the rod $AB$ is replaced by a uniform triangular lamina $ABC$ of weight $W$, with $A$ resting against a rough vertical wall and $AB$ horizontal, as shown in the diagram. It is held in this position by means of a light inextensible string attached at $C$. The other end of the string is attached to a fixed point $D$ vertically above $A$ such that $DC$ with $AC$. The lamina and the string lie in one vertical plane perpendicular to the wall. Using a similar method to that used in (a) find, in terms of the weight $W$, the magnitude of the tension in the string and the total reaction at $A$. Find also the coefficient of friction between $A$ and the wall.

[1, 1, 3, 2, 3 marks]
8. Three identical smooth spheres $A$, $B$ and $C$ of mass 1 kg, lie at rest in a straight line $ABC$ on a smooth horizontal plane with $AB = BC = 10$ m. At time $t = 0$, the sphere $A$ and $C$ are projected towards $B$ with the velocities $1 \text{ ms}^{-1}$ and $0.5 \text{ ms}^{-1}$ respectively. When $A$ collides with $B$, the speed of $A$ becomes $0.2 \text{ ms}^{-1}$ in the same direction. It can be assumed that all impacts have the same coefficient of restitution.

i) By considering the first impact between $A$ and $B$ find the coefficient of restitution.

ii) Find the time $t$ when $B$ collides with $C$.

iii) Calculate the impulse that $B$ exerts on $C$ in the second impact, that is the first one between $B$ and $C$.

[4, 3, 3 marks]

9. An light elastic string has a natural length 2 m and modulus of elasticity 150 N. One end of the string is attached to a fixed point, and a particle of mass 3 kg hangs from the other end.

i) Find the extension of the string when the particle is in equilibrium.

ii) The particle is pulled down a further 10 cm and then released. Find its speed as it passes through the equilibrium position.

iii) Find the acceleration of the particle at a general position, in terms of the displacement, $x$, measured from the equilibrium position. Hence show that it moves with SHM.

iv) What is the amplitude of the resulting oscillations?

v) State where maximum acceleration occurs and find its magnitude.

vi) Find the period of the subsequent oscillations.

[1, 3, 3, 1, 1, 1 marks]
10. A light inextensible string $AB$ of length $2l$ has a particle of mass $m$ attached to its mid-point $C$. The ends $A$ and $B$ of the string are fastened to two fixed points with $A$ a distance $l$ vertically above $B$. With both parts of the string taut, the particle describes a horizontal circle about the line $AB$ with constant angular speed $\omega$. If the tension in $CA$ is three times that in $CB$:

i) show, by resolving forces vertically, that the tension in the lower string is equal to the weight;

ii) express the radius of the circle, described by the particle, in terms of $l$;

iii) using $F = ma$ show that $\omega = 2\sqrt{\frac{g}{l}}$.

[3, 2, 5 marks]