Directions to candidates

The marks carried by each question are shown.
The total number of marks for all questions in the paper is 100.
Graphical calculators are NOT allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.
Where necessary take $g=10\text{ms}^{-2}$
1. Referred to a fixed origin $O$ and unit perpendicular vectors $\mathbf{i}$ and $\mathbf{j}$, the position vectors of the vertices of a triangle $ABC$ are: $\mathbf{OA} = (-15\mathbf{i} - 20\mathbf{j})$, $\mathbf{OB} = 25\mathbf{i}$, $\mathbf{OC} = (-7\mathbf{i} + 24\mathbf{j})$. $D$ is the midpoint of side $BC$ in triangle $ABC$. (See diagram.)

(a) Find the lengths of $\mathbf{OA}$, $\mathbf{OB}$ and $\mathbf{OC}$, and show that they are equal.
(b) Prove that: $5(\mathbf{AB} + \mathbf{AC}) = 16 \mathbf{AO}$.
(c) Determine the position vector of $D$ and hence show that $A$, $O$, $D$ are collinear.

Three forces, $\mathbf{F}_{OA}$, $\mathbf{F}_{OB}$ and $\mathbf{F}_{OC}$, all of magnitude 100N, act at $O$ in the directions $\mathbf{OA}$, $\mathbf{OB}$, and $\mathbf{OC}$ respectively.

(d) Show that $\mathbf{F}_{OA} = 4(-15\mathbf{i} - 20\mathbf{j})$N, and find similar expressions for $\mathbf{F}_{OB}$ and $\mathbf{F}_{OC}$.
(e) Find the vector resultant of forces $\mathbf{F}_{OA}$, $\mathbf{F}_{OB}$ and $\mathbf{F}_{OC}$ and prove that the line of action of this resultant passes through point $D$. [2, 3, 5, 3, 4 marks]

2. Two rings, $A$ and $B$, each of mass 1kg, are connected by a light, elastic string whose modulus of elasticity is 10N. Ring $A$ is slung on a rough, fixed, horizontal rod while $B$ can slide on a smooth inclined rod. The rods, which lie in one vertical plane, are firmly joined together at $O$ and make 120° with each other. When the system is in limiting equilibrium, with $A$ at a higher level than $B$, the distances $AO$ and $OB$ are both 1m. (See diagram)

(a) Determine the angle the string makes with each rod and show that $AB = 3^{1/2}$ m.
(b) Draw diagrams showing the forces acting on $A$ and $B$. Show that the three forces acting on $B$ make equal angles with each other.
(c) By considering the equilibrium of $B$ find the tension in the string.
(d) Determine the natural length of the elastic string giving your answer in surd form.
(e) Find the normal reaction of the horizontal rod on $A$ and determine the coefficient of friction, in surd form, between $A$ and the horizontal rod. [1, 3, 2, 3, 5 marks]
3. A uniform rectangular lamina \(ABCD\), of weight 64N, has side \(AB = 120\) cm and side \(AD = 32\) cm. The lamina is in equilibrium with its corner \(A\) freely hinged to a fixed point and its other corner \(C\) in contact with a smooth plane inclined at \(\tan^{-1}(15/8)\) to the horizontal. The lamina, which lies in a vertical plane with a line of greatest slope of the inclined, has side \(AB\) topmost and horizontal. \(O\) is the midpoint of \(AB\). (See diagram.)

(a) Explain carefully, with reference to the ‘Three-Force Result’ and other pertinent theory, why the forces which keep the lamina in equilibrium are parallel to the sides of triangle \(COB\).

(b) By means of a diagram show the lines of action of the forces acting on the lamina and determine their magnitudes.

The lamina is now no longer supported by the inclined plane but is kept in equilibrium in the same position as before by means of a light vertical string attached to the corner \(B\).

(c) Determine the new magnitude and direction of the reaction of hinge at \(A\) now.

[6, 3, 3 marks]

4. A uniform rod \(AB\), of length 120 cm and weight 120 N, is freely hinged to a wall at its end \(A\). The rod has a moveable ring, of weight 80 N, slung at a point \(C\), \(x\) cm away from \(A\). The rod is kept in equilibrium in a horizontal position by means of a light rope making \(30^0\) to the horizontal. An end of this rope is attached to the rod at \(D\), 80 cm from \(A\), and the other end fixed to a point on the wall vertically above \(A\). (See diagram.)

(a) Draw a diagram showing all the forces acting on the system of rod plus ring.

(b) By taking moments about a suitable point find the tension in the string in terms of \(x\).

(c) Find expressions, in terms of \(x\), for the horizontal and vertical components of the reaction at \(A\).

(d) Give the value of the resultant reaction at \(A\) when this, for a particular value of \(x\), is directed horizontally.

[2, 3, 4, 2 marks]
5. **ABCD** is a rectangle with side \( AB = 20 \text{ cm} \) and side \( AD = 10 \text{ cm} \). Four forces, coplanar with the rectangle, act at three of its vertices. The magnitudes and directions of these forces are as shown in the adjoining diagram.

(a) Show that the sum of the clockwise moments of the forces about \( C \) is given by:
\[
(600 - 6P) \text{ N cm.}
\]
(b) Find the values of \( P, X \) and \( Y \) in the three separate cases when the resultant of the four forces is:

(i) equilibrium;
(ii) a clockwise couple of moment 120 N cm;
(iii) a force of 10N acting at \( C \) in the direction \( \text{DC} \).

[3, 4, 4, 4 marks]

6. At time \( t = 0 \) seconds, the drivers of two cars \( A \) and \( B \), which are travelling along a narrow straight road in opposite directions with constant speeds of \( 16 \text{ ms}^{-1} \) and \( 20 \text{ ms}^{-1} \) respectively, realise they are on a collision course. At this moment the distance apart between the cars is 116m. The driver of car \( A \) applies his brakes at \( t = 2 \text{ s} \) and the car is uniformly retarded and comes to rest three seconds later, at \( t = 5 \text{ s} \).

(a) Draw a velocity – time graph for the last 5 seconds of motion of car \( A \). Include all pertinent data in your work.
(b) Find the distance travelled by the car in this time and the value of its retardation in \( \text{m s}^{-2} \) in its last 3s of its motion.

The driver of car \( B \) applies the brakes of his car at \( t = T \text{ s} \) seconds and the car is uniformly retarded to come to rest simultaneously with car \( A \) while barely avoiding a collision.

(c) Draw a velocity – time graph for car \( B \) for its last 5 seconds of its motion including all pertinent data.
(d) Determine the value \( T \) and that of the retardation of car \( B \) in \( \text{m s}^{-2} \).
(e) Find the time, apart from \( t = 5 \text{ s} \), when the cars \( A \) and \( B \) are moving with the same speeds in their last 5 seconds of their motion.

[2, 3, 2, 6, 2 marks]
7. Two small blocks, $A$ and $B$, of masses 6 kg and 4 kg respectively, are connected by a light inextensible string. Block $A$ lies on the smooth surface of a plane inclined at $30^0$ to the horizontal, while $B$ is on the rough surface of a horizontal plane. The connecting string passes over a small, smooth pulley $P$ fixed on the edge where the two planes meet. Initially, both blocks and their taut connecting string are in a vertical plane, which cuts the inclined plane on a line of greatest slope. The coefficient of friction between $B$ and the horizontal plane is $\frac{1}{4}$. When both blocks are simultaneously released from rest $B$ lies 180 cm away from $P$. (See diagram.)

(a) Draw diagrams showing the forces acting on each block together with their acceleration.

(b) By applying Newton’s Second Law of Motion to $A$, show that, while the two blocks are in motion, the tension $T$, in N, and the acceleration $f$, in $\text{ms}^{-2}$, are connected by:

$$30 - T = 6f.$$ 

(c) Apply Newton’s Second Law, vertically and horizontally to $B$, while both blocks are in motion. Hence prove that the common acceleration of the blocks is $2\text{ ms}^{-2}$.

When the two blocks have both traversed a distance of 1m, block $A$ reaches the end of the inclined and stops. Block $B$ continues in its motion.

(d) Show that $B$ just reaches the pulley $P$ before it stops. 

[3, 3, 4, 2, 4 marks]

END OF TEST