Directions to candidates

The marks carried by each question are shown.
The total number of marks for all questions in the paper is 100.
Graphical calculators are NOT allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.
Where necessary take g = 10ms$^{-2}$
1. \(ABC\) is a triangle. Vector \(\mathbf{AB} = 6\mathbf{x}\) and vector \(\mathbf{AC} = 6\mathbf{y}\). \(D\) and \(E\) are the midpoints of sides \(AB\) and \(AC\) respectively. (See diagrams)

(a) Show that \(\mathbf{BE} = 3\mathbf{y} - 6\mathbf{x}\) and find a similar expression for \(\mathbf{CD}\).

Referred to an origin \(O\) the position vector of \(A\) is given by \(\mathbf{OA} = \mathbf{a}\). \(G\) is a point inside triangle \(ABC\) such that \(\mathbf{AG} = 2\mathbf{x} + 2\mathbf{y}\).

(b) Show that the position vector of \(B\) is \(\mathbf{OB} = \mathbf{a} + 6\mathbf{x}\) and find similar expressions for the position vectors of \(C\) and \(G\).

(c) Prove that \(\mathbf{OA} + \mathbf{OB} + \mathbf{OC} = 3\mathbf{OG}\).

(d) Show that \(3\mathbf{GE} = \mathbf{BE}\) and that \(3\mathbf{GD} = \mathbf{CD}\).

Hence prove that \(G\) is the centroid of triangle \(ABC\), that is \(G\) is the point where, the so-called medians, \(BE\) and \(CD\) meet.

Referred to the origin \(O\) and unit, perpendicular vectors \(\mathbf{i}\) and \(\mathbf{j}\), two vertices of a triangle have position vectors \((2\mathbf{i} + 4\mathbf{j})\) and \((7\mathbf{i} + 8\mathbf{j})\). The third vertex is at \(O\).

(e) Determine the position vector of the centroid of this triangle and find its distance from the origin. [2, 2, 3, 4, 3 marks]

2. The diagram shows two identical particles \(A\) and \(B\), of mass 1 kg, joined by a taut, light string. \(A\) is on a rough horizontal plane, while \(B\) lies on a rough plane which is inclined at \(\theta = \tan^{-1}(3/4)\) to the horizontal. The string passes over a small, smooth pulley \(P\) fixed on the line of intersection of the planes. This line is normal to both sections of the string, \(AP\) and \(BP\). The system is in limiting equilibrium with the coefficient of friction between both particles and planes being \(\mu\).

(a) Draw two separate diagrams showing the directions of the forces on \(A\) and on \(B\).

(b) By considering the limiting equilibrium of \(A\), and resolving forces in two directions, show that the tension, \(T\), in the string is given by \(T = (10\mu)\) N.

(c) Show that \(\cos \theta = 4/5\) and prove that the normal reaction of the inclined plane on \(B\) is 8 N. Derive a second equation relating \(T\) and \(\mu\).

(d) Determine the value of \(\mu\). [4, 3, 4, 2 marks]
3. A rigid body in equilibrium is acted upon by three forces \( P \), \( Q \), and \( R \). The lines of action of the forces \( P \) and \( Q \) meet at a point \( O \).

(a) What can you say regarding the line of action of force \( R \)?

A uniform rod \( AB \), of length 2 m and weight 10N, is freely hinged to a vertical wall at its end \( A \). It is kept in equilibrium by means of a light elastic string attached to its lower end \( B \), and to a fixed point \( C \), where \( BC = 1 \) m. The modulus of elasticity of the string is 20 N. The point \( D \) is on the wall, vertically below \( A \), such that triangle \( ADB \) is equilateral with side \( DB \) in line with string \( BC \). \( O \) is the midpoint of \( DB \). (See diagram)

(b) Explain carefully why triangle \( ADO \) has its sides parallel to the three forces which keep the rod \( AB \) in equilibrium. (That is triangle \( ADO \) is a Triangle of Forces.)

(c) Find the magnitude of the reaction at \( A \) and the tension in the string.

(d) Determine the natural length of the elastic string. \([1, 4, 4, 2 \text{ marks}]\)

4. A uniform ladder \( AB \), of weight 100 N, is in equilibrium with its lower end \( A \) on rough horizontal ground and with end \( B \) in contact with a smooth vertical wall. \( A \) is 80 cm away from the wall and \( B \) is 200 cm above the ground. The ladder is in a vertical plane normal to the wall and has a load of 100 N hung at a point \( C \), where \( AC = (\frac{3}{4})AB \).

(a) Draw a diagram showing the lines of action and directions of the forces keeping the system of ladder plus load in equilibrium.

(b) By taking moments about a suitable point find the reaction of the wall on the ladder.

(c) Find the normal and frictional reaction of the ground on the ladder at \( A \).

(d) Show that the resultant reaction at \( A \) makes an angle of \( \tan^{-1}(4) \) with the horizontal. \([2, 4, 3, 1 \text{ marks}]\)

5. \( A \), \( B \), and \( C \) are three points in a line with \( AB = BC = 2 \) m. Forces \( P \) and \( Q \) act at \( A \) perpendicular and along the direction \( AC \) respectively. A force \( R \) acts at \( B \) in the direction opposite to that of \( P \), and a force of magnitude 100 N acts at \( C \) making angle \( \theta = \tan^{-1}(3/4) \) with \( CA \) and angle \( (90-\theta) \) with \( P \). (See diagram)

Find \( P \), \( Q \), and \( R \) in the separate cases when the resultant of the system of forces is:

(i) equilibrium;

(ii) a clockwise couple of magnitude 40 Nm;

(iii) a force of magnitude 20 N acting at \( C \) in the direction of \( P \). \([6, 6, 6 \text{ marks}]\)
6. Two experimental, solar-powered cars, \(P\) and \(Q\), are driving alongside each other on either side of the central strip of a straight track. At time \(t = 0\) seconds the cars' fronts are both alongside a point \(A\) on the central strip and then at time \(t = 36\) s, the cars come to a stop together when their fronts are both alongside another point \(B\) on the central strip.

Car \(P\) is moving with constant velocity of \(12\) m s\(^{-1}\) after \(t = 0\) s, until, at \(t = 12\) s, it decelerates uniformly to come to rest, at \(t = 36\) s, alongside \(B\).

(a) Draw the velocity-time graph of the last 36 seconds of \(P\)'s motion.
(b) Determine the distance \(AB\).

Car \(Q\) is moving with constant velocity of \(9\) m s\(^{-1}\) after \(t = 0\) s, until at \(t = T\) seconds, it decelerates uniformly to come to rest, at \(t = 36\) s, alongside \(B\).

(c) Draw the velocity-time graph of the last 36 seconds of \(Q\)'s motion.
(d) Deduce the value of \(T\) and find the decelerations of both \(P\) and \(Q\) as they come to a stop.
(e) Find the time, other than \(t = 36\) s, when the cars happen to have the same velocity.

\[2, 2, 2, 7, 3\] marks

7. A small block, \(A\), of mass 3 kg, lies at rest at the centre of the smooth surface of a long, horizontal table. It is attached, by light inextensible strings, to similar blocks, \(B\) and \(C\), of masses 4 kg and 3 kg respectively, which hang freely at opposite sides of the table. The strings pass over smooth pulleys at either end of the table. When the system is released from rest, with the three blocks and pulleys all in one vertical plane, \(B\) is 50 cm above the floor while \(C\) is 60 cm below the table. (See diagram)

(a) Draw a diagram showing both the directions of the forces acting on \(A\) and its acceleration while all 3 blocks are still in motion. Draw similar diagrams for \(B\) and \(C\).
(b) Apply Newton's Second Law of Motion to each of the blocks \(A\), \(B\), and \(C\), while all 3 blocks are in motion. Hence show that the common acceleration of the blocks is \(1\) m s\(^{-2}\).
(c) Find the speed with which \(B\) hits the ground.

When \(B\) hits the ground it stops immediately while \(A\) and \(C\) continue their motion.

(d) Determine the retardation of \(A\) and \(C\), while \(B\) is at rest on the ground.
(e) Show that \(C\) will come momentarily to rest just as it reaches the pulley above it.

\[4, 5, 2, 5, 2\] marks

END OF QUESTIONS